

New Method of Computation of Electrical Y-Matrix for Arbitrarily Connected Multielement SAW Devices

Sergei M. Balashov, Renato Cechetti Pinto, Clovis M. Cabreira, and Célio Antonio Finardi

Abstract—We treat surface-acoustic wave (SAW) devices as an arbitrary multielement system. To describe it, a modified form of the chain matrix (E -matrix) is introduced. The E -matrix has a property of turning the calculation of the transfer matrix of this system into a simple multiplication procedure. This allows one to perform computation of electrical response of almost any type of SAW device since the total transfer matrix could be reduced to a Y -matrix. Examples and simulation results confirming the method described are shown.

Index Terms—Cascading algorithm, filters, multichannel device, surface-acoustic waves.

I. INTRODUCTION

CURRENTLY, surface-acoustic wave devices (SAWD's) are widely used in the design of various RF systems in the range of 30 MHz–2.5 GHz. SAWD's are used as resonators, passband filters, various delay lines, convolvers, etc. The response of any SAWD could be obtained as a result of cascading of its multiport building blocks (BB's) (transducers, delay lines, reflectors, etc.). Thus, it is important to have efficient tools to calculate the electrical response of sets of these elements arbitrarily connected. General methods of net analysis could be used to calculate the admittance matrix (Y -matrix) of such structures, but a large number of BB's and various topology implementations, together with a great number of models used to describe each BB, make this general approach impractical for the design of SAWD's in real computer-aided design (CAD) systems. The main difference between a SAW BB and a normal N -pole element is that energy transfer from one BB to another exists in the form of a SAW, but not as an electrical current. It happens that in all types of SAWD's it is possible to specify subsets of BB's which make the chains [acoustic channels (AC's)], where SAW energy is transferred from one BB to another. Several types of BB's can transform acoustic energy from one AC to another and others can transform it to an electrical one.

Recently, several attempts were made to create a general method to cascade arbitrary SAW BB's [1], [2], but until now the algorithm from [1] could be implemented only for design of a SAWD with one AC, and the method from [2]

imposes some restrictions on the model used to represent the BB. In this paper, we present the algorithm which allows one to calculate the Y -matrix of a SAWD, consisting of an arbitrary number of AC's. Arbitrary number of BB's, transferring SAW energy from one channel to another, could be introduced and no restrictions on the models used to represent a BB should be applied. The method presented, having the same procedure of cascading for all SAWD structures, could be effectively used in the design of practically all types of SAW RF components.

II. SAW DEVICE AS A MULTIELEMENT SYSTEM

Consider the system which has a total of Q BB's placed in M AC's. This system can represent almost any SAW device (see Fig. 1). We divide all BB's into two different types: pure acoustic BB's, which do not have an electrical terminal, and electroacoustic BB's (EABB's), which have it. The first group consists of delay lines, multistrip couplers (MSC's), reflectors, etc. The second one consists of various types of interdigital transducers (IDT's). In Fig. 1, for example, the system has a total of N EABB's. To describe this system, we determine the output and input vectors as follows:

$$A^t = [\phi_1, \phi'_1, \dots, \phi_M, \phi'_M, I_1, \dots, I_N] \quad (1)$$

$$B^t = [\mu_1, \mu'_1, \dots, \mu_M, \mu'_M, V_1, \dots, V_N] \quad (2)$$

where superscript t signifies the operation of the matrix transpose. Components of (1) and (2) are defined in Fig. 1. The chosen order of components is the most convenient for the process of BB cascading. The electroacoustic chain matrix C characterizes the system completely, connecting columns A and B as follows:

$$A = C \cdot B. \quad (3)$$

However, normally SAWD are specified by network measurements. To shrink C to the Y -matrix of the system it is necessary to take advantage of acoustic boundary conditions. Conditions imposed by the presence of acoustic absorbers on the borders of the crystal give

$$\phi'_i = \mu_i = 0, \quad i = 1, \dots, M. \quad (4)$$

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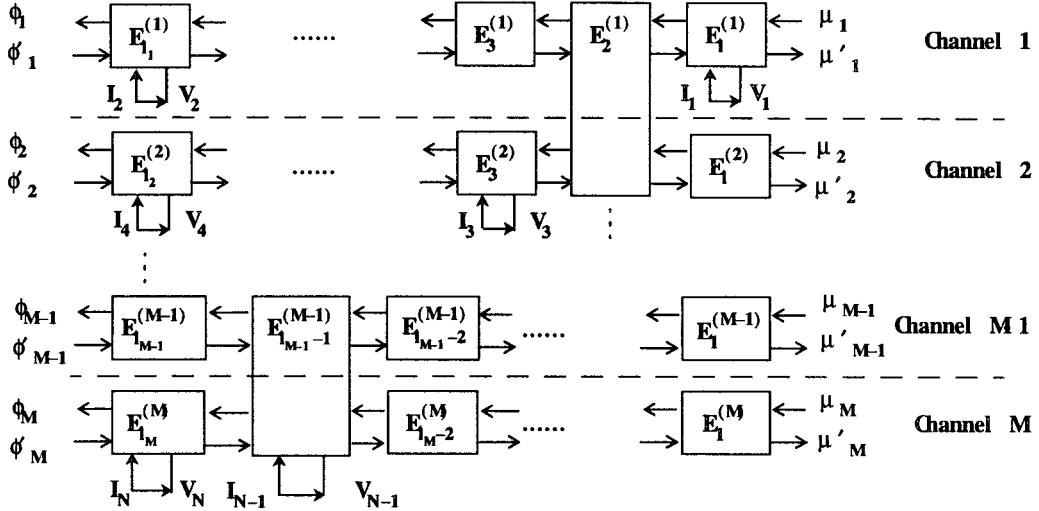


Fig. 1. Graphic representation of SAWD structure consisting of Q BB and M AC. μ_j , μ'_j , and ϕ_j , ϕ'_j are SAW amplitudes on input and output acoustic ports, respectively. $E_j^{(i)}$ are BB's. $E_1^{(1)}, E_2^{(1)}, E_3^{(2)}, E_{l_2}^{(2)}$, and $E_{l_M}^{(M)}$ are one channel EABB's. $E_2^{(1)}$ is a pure acoustic multichannel BB. $E_{l_{m-1}-1}^{(M-1)}$ is an electroacoustic multichannel BB. I_i and V_i ($i = 1, \dots, N$) are currents and voltages. Note that only the EABB takes part in enumeration of I_i and V_i .

By substituting (4) into (3), one can easily obtain the Y -matrix of the system by elimination of acoustic variables in the form

$$[I_1, \dots, I_N]^t = Y \cdot [V_1, \dots, V_N]^t \quad (5)$$

where

$$Y = D - F(G)^{-1}H \quad (6)$$

$$D_{i,j} = C_{2M+i,2M+j}, \quad i, j = 1, \dots, N \quad (7)$$

$$F_{i,j} = C_{2M+i,2j}, \quad i = 1, \dots, N; j = 1, \dots, M \quad (8)$$

$$G_{i,j} = C_{2i,2j}, \quad i, j = 1, \dots, M \quad (9)$$

$$H_{i,j} = C_{2i,2M+j}, \quad i = 1, \dots, M; j = 1, \dots, N. \quad (10)$$

Calculation of the Y -matrix completes the analysis of the SAWD because further transforms could be done by methods of net theory.

III. E-MATRIX CONSTRUCTION

The C -matrix of the SAWD should be calculated on the basis of properties of each BB. To do this, we introduce an extended chain matrix (E -matrix) of each BB, which is based on the classic chain matrix (T -matrix) [3] representation. The E -matrix of a given BB will be obtained by distributing over its elements the elements of the T -matrix. The position which a given element of the T -matrix occupies inside the E -matrix depends on the position of the BB inside the SAWD structure. This procedure will allow one to substitute BB cascading by multiplication of E -matrices for arbitrary topology.

To start the process of E -matrix construction, let us enumerate all EABB's in a given SAWD from the first to the last channel and from the right to the left EABB (see Fig. 1 caption). Consider that the total number of EABB's is N and the BB under study has the number n_0 in the set of EABB's. Let us build the chain matrix of IDT, which occupies K adjacent AC's (see Fig. 2), as an example of the most general

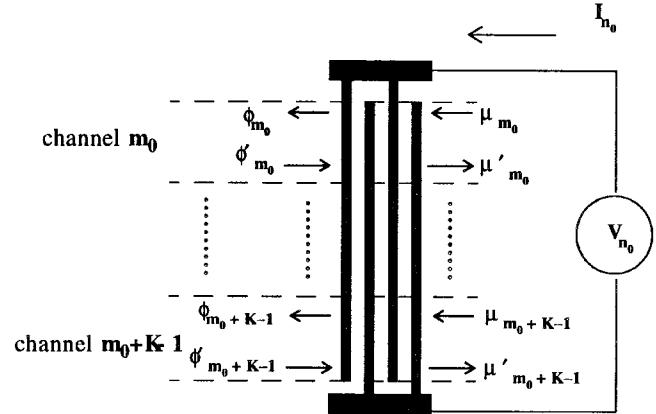


Fig. 2. Topology of multichannel EABB. V_{n_0} is the applied voltage source, and I_{n_0} is the electric current. The upper channel of a given IDT is placed in channel m_0 of the SAWD.

multichannel EABB. Its chain matrix $T = \{t_{ij}\}$ is determined as follows:

$$\begin{bmatrix} \phi_{m_0} \\ \phi'_{m_0} \\ \dots \\ \phi_{m_0+K-1} \\ \phi'_{m_0+K-1} \\ I_{n_0} \end{bmatrix} = T \begin{bmatrix} \mu_{m_0} \\ \mu'_{m_0} \\ \dots \\ \mu_{m_0+K-1} \\ \mu'_{m_0+K-1} \\ V_{n_0} \end{bmatrix} = \begin{bmatrix} t_{11} & \dots & t_{1,2K+1} \\ t_{21} & \dots & t_{2,2K+1} \\ \dots & \dots & \dots \\ t_{2K+1,1} & \dots & t_{2K+1,2K+1} \end{bmatrix} \begin{bmatrix} \mu_{m_0} \\ \mu'_{m_0} \\ \dots \\ \mu_{m_0+K-1} \\ \mu'_{m_0+K-1} \\ V_{n_0} \end{bmatrix} \quad (11)$$

where m_0 is the number of upper AC occupied by this BB inside the SAWD. All the other variables are determined in

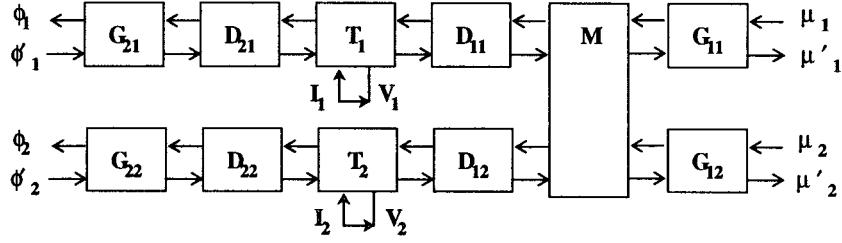


Fig. 4. SAW two-port MSC coupled resonator. Gratings are 9.6-mm long. Period is $12 \mu\text{m}$. Transducers have five fingers on a YZ-LiNbO₃ substrate. Aperture is 50 wavelengths. Coupling coefficient is 4.3 cm^{-1} . Parasitic electrodes series resistance is 11Ω . D_{11} and D_{12} give phase shift of 0.25π . D_{21} and D_{22} give 9.89π . MSC is $90\text{-}\mu\text{m}$ long, with $5\text{-}\mu\text{m}$ strips.

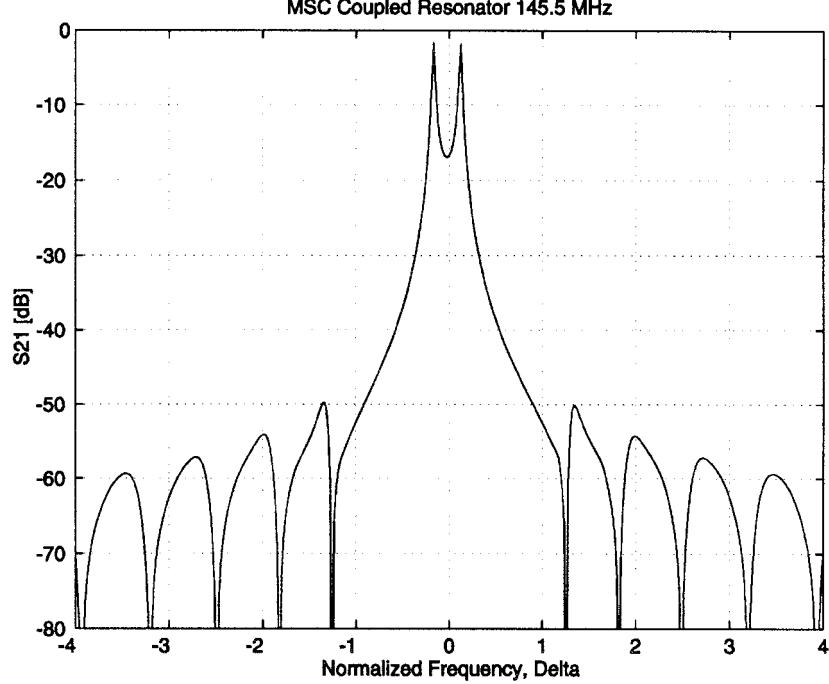


Fig. 5. Electrical response of the SAW two-port MSC coupled resonator. Insertion loss is calculated for an unmatched condition in a $50\text{-}\Omega$ net.

channel. This position is described by the lower index of the E -matrix of the BB.

Equations (24) and (25) cover all possible cases and should be used iteratively until $m = M$. The result matrix C_m is the C -matrix of the given SAWD. The validity of the process described could be proved by applying it to all possible pairs of BB's and further grouping of an arbitrary SAWD structure into new blocks, each containing several BB described before. Here, for simplicity, we consider the situation when only one BB transforms the SAW energy from one channel to another. A more general case could be treated using the same approach.

V. SIMULATION RESULTS

The proposed method of Y -matrix determination could be effectively applied to various structures, starting from a simple single channel to complex multichannel devices. To illustrate this method we consider three structures here:

- 1) SAW two-port multistrip coupled resonator [7];
- 2) single-channel two-port SAW resonator [7];
- 3) three-channel SAW filter with channels connected electrically [8].

For the first and the second devices the T -matrices of the BB were calculated using a Meason equivalent-circuit model from [7]. T -matrices of the third device were calculated on the basis of the coupling-of-modes (COM) model [6]. Model parameters used are listed in the captions of the figures with the block structures of these devices. Parameters are the same as in references.

The first device, being the most general representative of a multichannel SAWD (Fig. 4), has its C -matrix construction described in details in the Appendix. Final insertion loss of this device is shown in Fig. 5. The cavities are overcoupled and resonance splits into two peaks. The result shown is in a good agreement with that from [7].

The block diagram of the second device is shown in Fig. 6. The device has a single channel, with its C -matrix easily obtained in the form

$$C = E_{G_2} \cdot E_{D_{12}} \cdot E_{T_2} \cdot E_{D_2} \cdot E_{T_1} \cdot E_{D_{11}} \cdot E_{G_1}. \quad (26)$$

The insertion loss of this device is shown in Fig. 7. For the third device (Fig. 8) we can write

$$C = E_{T_6} \cdot E_{D_3} \cdot E_{T_5} \cdot E_{T_4} \cdot E_{D_2} \cdot E_{T_3} \cdot E_{T_2} \cdot E_{D_1} \cdot E_{T_1}. \quad (27)$$

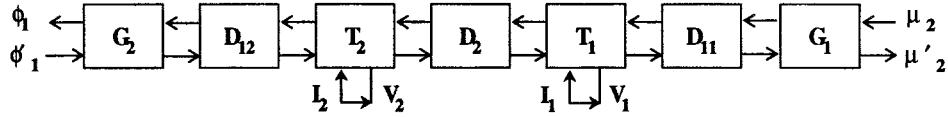


Fig. 6. SAW two-port resonator. Gratings G_1 and G_2 are 9.6-mm long. Period is $12 \mu\text{m}$. Transducers T_1 and T_2 have five fingers on a YZ-LiNbO₃ substrate. Aperture is 50 wavelengths. Coupling coefficient is 4.5 cm^{-1} . Parasitic electrodes series resistance is 11Ω . Delay lines D_{11} and D_{12} give phase shift of 0.25π ; D_2 gives 9.98π .

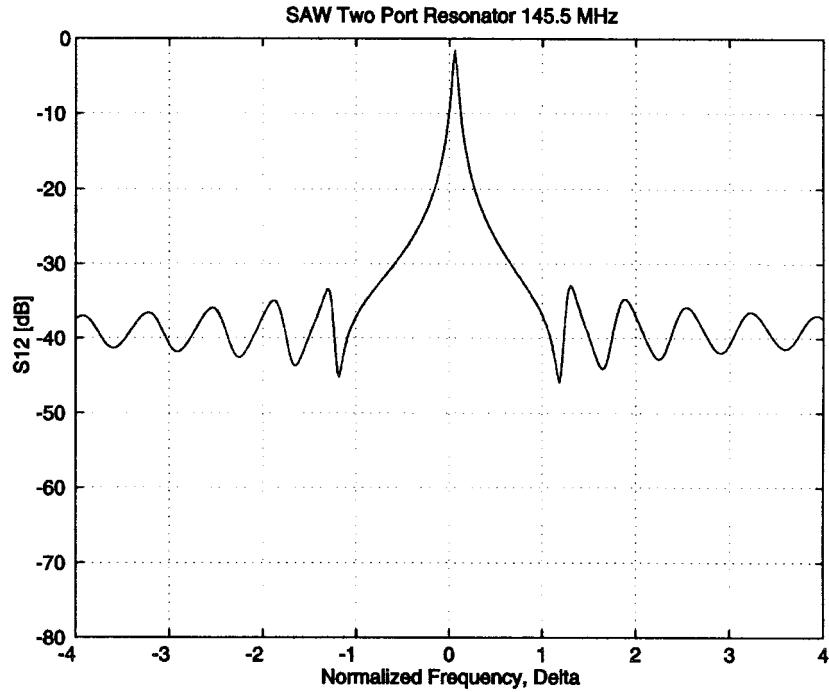


Fig. 7. Electrical response of the SAW two-port resonator. Insertion loss is calculated for an unmatched condition in a 50Ω net.

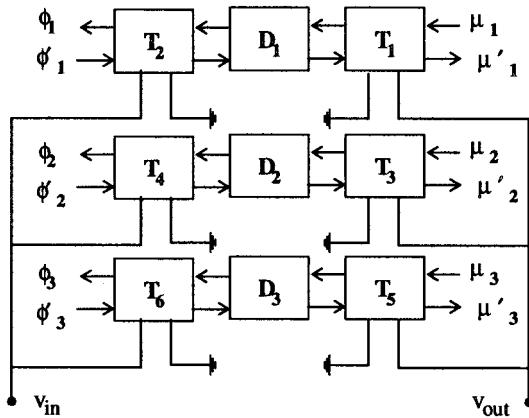


Fig. 8. Three tracks SAW filter. Numbering of transducers follows the order of the BB inside the EABB set. Length l of each delay line differs by $\lambda/3$: $l_1 = 2\lambda$, $l_2 = l_1 - \lambda/3$, $l_3 = l_2 - \lambda/3$ where λ is the transducer period. Transducers are 80 periods long, center frequency is 71 MHz, acoustic aperture is 550λ , metallization height is 6000 Å. ST-X Quartz substrate, COM parameters: $\kappa = 14.29 \times 10^{-3}$, $\alpha = 83.02 \times 10^{-5}$, $\gamma = 2 \times 10^{-4}$. Input and output transducers are connected in parallel.

Use of (6)–(10) in this case leads to a Y -matrix which has a dimension 6×6 , in correspondence with the number of electric ports in this filter. The standard method of network analysis allows one to take into account electrical connections and shrink this matrix to the size 2×2 . Final insertion loss

of this filter, shown in Fig. 9, is in a good agreement with the results of [8].

Note that to characterize three structures which have quite a different topology, the same algorithm was used without any changes because the E -matrices contain almost all necessary information about SAWD topology from the very beginning. For E -matrices having many zero elements, the special algorithms of the matrices multiplication could be used to dramatically decrease the computation time.

VI. CONCLUSIONS

A modified method for calculation of the electrical Y -matrix for arbitrary SAW devices has been presented. It is based on the E -matrix introduction. This matrix incorporates topological information about the SAWD structure, which simplifies the process of cascading of such matrices. The cascading process for this matrices was described. Three examples with simulation results were presented. The algorithm is extremely suitable for use in any CAD system because it allows one to cascade BB's of a practically arbitrary SAWD without any restrictions imposed on the model used to represent its BB.

APPENDIX

Consider a multistrip coupled resonator shown in Fig. 4. We will present E -matrices and cascading process for this device

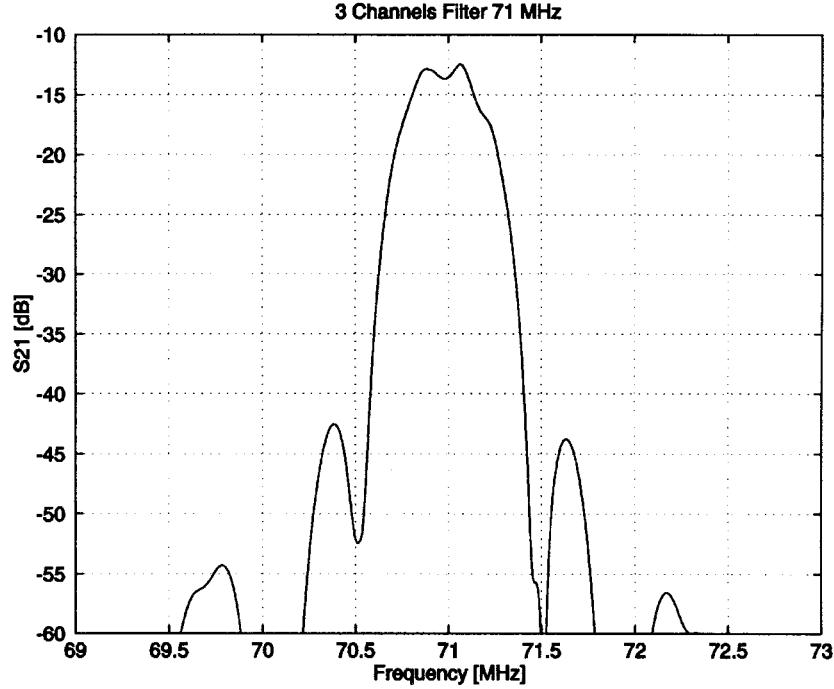


Fig. 9. Electrical response of a three-channel SAW filter. Insertion loss is calculated for matched condition in a 50Ω net.

as an example of how one can apply the method proposed to a real SAWD. From the analysis of the BB structure of this filter, it is possible to identify: 1) two single-channel EABB's: T_1 and T_2 , which gives $N = 2$; 2) one pure acoustic multichannel BB: M ; and 3) eight pure acoustic single-channel BB: four gratings: G_{11} , G_{12} , G_{21} , G_{22} , and four delay lines D_{11} , D_{12} , D_{21} , and D_{22} , placed into two AC's, which means that $K = 2$. Following the method presented, one should start with (16) and generate 6×6 unit matrices for each BB as E -matrices. Since T_1 is in the first channel, $m_0 = 1$. It is also the first EABB, which means that $n_0 = 1$. Using of (17)–(21) with these parameters gives the E -matrix for T_1 in the following form:

$$E_{T_1} = \begin{bmatrix} t_{11} & t_{12} & 0 & 0 & t_{13} & 0 \\ t_{21} & t_{22} & 0 & 0 & t_{23} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ t_{31} & t_{32} & 0 & 0 & t_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (\text{A.1})$$

Since T_2 is situated in the second channel, $m_0 = 2$. If it is the second EABB, then $n_0 = 2$. Using these values, we get

$$E_{T_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & t_{11} & t_{12} & 0 & t_{13} \\ 0 & 0 & t_{21} & t_{22} & 0 & t_{23} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & t_{31} & t_{32} & 0 & t_{33} \end{bmatrix}. \quad (\text{A.2})$$

Note that since T_1 and T_2 are equivalent, they have the same T -matrix.

The multichannel BB is a MSC, thus it has only a T^A part, and only (17) with $K = 2$ should be used to construct its

E -matrix. Since $m_0 = 1$, we have

$$E_M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & 0 & 0 \\ m_{21} & m_{22} & m_{23} & m_{24} & 0 & 0 \\ m_{31} & m_{32} & m_{33} & m_{34} & 0 & 0 \\ m_{41} & m_{42} & m_{43} & m_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (\text{A.3})$$

All gratings have the same T -matrix. Matrices for G_{11} and G_{21} have $m_0 = 1$. For G_{21} and G_{22} $m_0 = 2$. Finally, for all of them, we have

$$E_{G_{11}} = E_{G_{21}} = \begin{bmatrix} g_{11} & g_{12} & 0 & 0 & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A.4})$$

$$E_{G_{21}} = E_{G_{22}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_{11} & g_{12} & 0 & 0 \\ 0 & 0 & g_{21} & g_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (\text{A.5})$$

E -matrices for delay lines could be easily constructed in a way similar to (A.4) or (A.5) with the additional condition $g_{11} = g_{22} = 0$. Cascading of E -matrices for this filter starts with (22), which gives

$$C_1 = E_{G_{21}} \cdot E_{D_{21}} \cdot E_{T_1} \cdot E_{D_{11}} \cdot E_M \cdot E_{G_{11}}. \quad (\text{A.6})$$

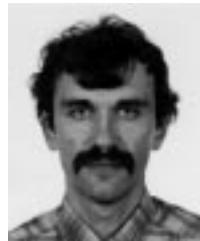
Once there is a multichannel BB between $E_{D_{12}}$ and $E_{G_{12}}$, it is necessary to use (25), which gives

$$C_2 = (E_{G_{22}} \cdot E_{D_{22}} \cdot E_{T_2} \cdot E_{D_{12}}) \cdot C_1 \cdot (E_{G_{12}}). \quad (\text{A.7})$$

C_2 is the final C -matrix of this filter. With this result, one can evaluate the electrical response using (6)–(10).

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